JPL TECHNICAL PEER REVIEW EVALUATION

DATE: 5/16/02

MANUSCRIPT TITLE: Solar Sail Dynamics and Control using a Boom Mounted Bus Articulated by a Bi-State Two-Axis Gimbal and Reaction Wheels

JOURNAL OR SOCIETY:
2002 AIAA Astrodynamics Specialist Conf.

AUTHOR: Edward Mettler and Scott R. Ploen EXT. 4-2071

REVIEWER: Dan Scharf EXT. 4-4795

COMMENTS:

One proposed control architecture for solar sail spacecraft consists of a reaction wheel on the spacecraft bus to control attitude and a motor between the solar sail and the bus—the motor controls the angle between them, generating center of pressure/center of mass offsets and hence solar torques. A motor, however, would introduce distubances into the sail, which is an extremely flexible structure. In addition the motor would have a large duty cycle as the spacecraft bus must be held at specified orientations with respect to the sail *and* the motor does not store energy.

The authors propose and demonstrate the feasiblity of an alternate method that eliminates the motor altogether. It is replaced by a locking gimbal. Center of pressure/center of mass offsets are generated by releasing the gimbal and using the reaction wheel. Since sail maneuvers consist of a see-saw-like bus motion, momentum is transfered in and out of the reaction wheel—a much more efficient method than just using a motor.

The main difficulty in this approach is deriving the non-trivial equations of motion. The derivation is done in a clean and straightforward manner using Lagrange's form of D'Alembert's principle. Numerous small effects are included such as the sail/bus gimbal not being located at the sail center of mass. After obtaining the truth model, it is linearized and simplified to obtain a model used for PD control design. With this controller the commanded torques and maneuver accuracy are shown to reasonable. The architecture is demonstrated.

The paper lucidly explains the new approach to solar sail control, accurately derives the applicable equations of motion and more than adequately demonstrates the approach's feasibility.

AUTHOR'S RESPONSE:

TECHNICAL REVIEWER SIGNATURE AND DATE:
Dan P Scharf 5/16/02

JPL TECHNICAL PEER REVIEW EVALUATION

DATE: 05/16/02

MANUSCRIPT TITLE: Solar sail dynamics and control using a boom mounted bus articulated by a bi-state two-axis gimbal and reaction wheels.

JOURNAL OR SOCIETY: AAS/AIAA Astrodynamics Specialist Conference Monterey, CA Aug. 2002

AUTHOR: E. Mettler, S. Ploen EXT. 4-2071

REVIEWER: Marco Quadrelli EXT. 4-7548

COMMENTS:

Bi-state two axis gimbal activation of solar sails is an innovative concept being explored today.
This paper contributes to the solar sail technology literature by analyzing the controlled dynamics of the sail under such novel type of activation.
This paper is a valuable contribution to the literature on sail dynamics and control.

AUTHOR'S RESPONSE:

TECHNICAL REVIEWER SIGNATURE AND DATE: Marco Quadrelli May 16, 2002

JPL 2378 6/91

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Date: 04/10/2002

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TITLE: Solar Sail Attitude Control by an Articulated Boom Mounted Bus Using a Bi-State Two-Axis Gimbal and Reaction Wheels

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NTR Title:	Solar Sail Attitude Control by an Articulated Boom Mounted Bus Using a Bi-State Two-Axis Gimb and Reaction Wheels
Evaluator:	KARINA L EDMONDS
Phone:	8183932827
Location:	202-216
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Date NTR Forwarded to Caltech for Caltech Patent Decision:	02/13/2002
Tech Brief Award recommendation:	
S/W Award recommendation:	
Patent decision by Caltech:	No
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Solar Sail Dynamics and Control using a Boom Mounted Bus Articulated by a Bi-State Two-Axis Gimbal and Reaction Wheels*

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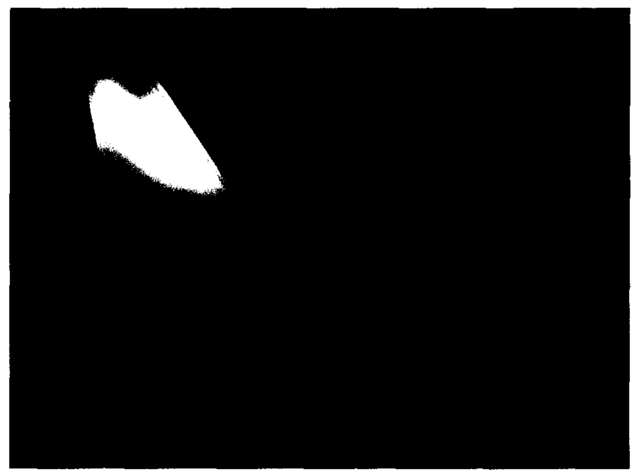
Picture 1: Sailcraft in Super-Synchronous Orbit



Picture 2: Fully Deployed Sailcraft

Abstract

In this paper we develop the coupled orbital/attitude dynamics of a multi-body solar sail spacecraft(sailcraft)under the gravitational attraction of a single central body. The solar sailcraft (Picture 1 and 2) is modeled as a chain of three rigid bodies (reaction wheel + bus/boom + solar sail). An additional aspect to our analysis is the introduction of a two axis bi-state gimbal into the multi-body model. The bi-state gimbal is simply a revolute joint that is either in a free or locked state: Here the rotational freedom between the bus/boom and the sail is assumed to be a bi-state hinge (See Figure 1 and 2). The derivation of the motion equations is based on a projected form of the Newton-Euler equations known as Lagrange's form of D'Alembert's principle [3], [4]. The resulting multi-body dynamic model captures (a) orbit-attitude and attitude-orbit coupling (b) internal reaction wheel momentum accumulation (c) gravity gradient effects (d) induced solar radiation forces and torques resulting from a flat plate sail geometry ([2], [6]) and (5) internal friction in the bi-state gimbal. We also demonstrate that the free-hinge motion equations can be readily modified to describe the system dynamics during periods when the bi-state gimbal is locked. The particular orbit of interest in our study is representative of the proposed NASA-JPL New Millennium Program Space Technology solar sail flight experiment that involves a 40 meter square sailcraft in a Super-Synchronous-Transfer-Orbit (SSTO). The SSTO is a highly elliptical Earth orbit with eccentricity = 0.8196, perigee = 8375 [km], apogee = 84482[km], and inclination = 35 [deg]. (See Figure 3) The multi-body dynamic model is then applied to

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demonstrate that a sail utilizing a simple bi-state gimbal concept can be used along with small standard reaction wheels to realize large commanded sail-sun incidence angles with a minimal momentum storage penalty, and without coupled structure control interaction (CSI) with the sail and booms. This is in contrast to performing a large angle slew via continuous actuation (e.g., via a stepper or servo motor) at the gimbal, that can induce coupled torque impulses and vibrations of the sail/booms, and thrust vector errors [9].

Introduction

The search for alternate methods of spacecraft propulsion has led to an increased interest in solar sailing technology [7]. However, there still remain a host of open issues in the area of solar sail dynamics and control that require additional investigation before solar sails become a viable alternative to traditional spaceflight architectures [1]. In this paper we develop the coupled orbital/attitude dynamics of a multi-body solar sail spacecraft under the gravitational attraction of a single central body. The solar sailcraft (Figure 1) is modeled as a chain of three rigid bodies (reaction wheel + bus/boom + solar sail). An additional aspect to our analysis is the introduction of a simple bi-state gimbal into the multi-body model. The bi-state gimbal is simply a revolute joint that is either in a free or locked state: Here the rotational freedom between the bus/boom and the sail is assumed to be a bi-state hinge (See Figure 1). The derivation of the motion equations is based on a projected form of the Newton-Euler equations known as Lagrange's form of D'Alembert's principle [3], [4]. The resulting multi-body dynamic model captures (a) orbit-attitude and attitude-orbit coupling (b) internal reaction wheel momentum accumulation (c) gravity gradient effects (d) induced solar radiation forces and torques resulting from a flat plate sail geometry ([2], [6]) and (5) internal friction in the bi-state gimbal. We also demonstrate that the free-hinge motion equations can be readily modified to describe the system dynamics during periods when the bi-state gimbal is locked. The particular orbit of interest in our study is representative of the proposed NASA-JPL New Millennium Program Space Technology solar sail flight experiment that involves a 40 meter square sailcraft in a Super-Synchronous-Transfer-Orbit (SSTO). The SSTO is a highly elliptical Earth orbit with eccentricity = 0.8196, perigee = 8375 [km], apogee = 84482[km], and inclination = 35 [deg]. (See Figure 3) The multi-body dynamic model is then applied to demonstrate that a sail utilizing a bi-state gimbal architecture can be used along with a reaction wheel to realize large commanded sail-sun incidence angles with a minimal momentum storage penalty, and without structure control interaction (CSI) with the sail and booms. This is in contrast to performing a large angle slew via continuous actuation (e.g., via a stepper motor) at the gimbal. Our proposed control strategy is based on a five-step maneuver: (1) The reaction wheel is commanded to create a small center of-mass to center-of-pressure (cm-cp) offset while the bi-state gimbal is free. This is accomplished by articulating the bus/boom relative to the solar-sail. (2) Once the appropriate cm-cp offset is attained the bi-state gimbal is locked and the system is accelerated toward the desired sun-incidence angle via solar radiation torques. (3) At an appropriate switching time the bi-state gimbal is again unlocked and the wheel is used to reverse the sign of the cm-cp offset (by again articulating the bus-boom relative to the sail) and hence reverse the direction of the incident solar radiation torque. As a result of this maneuver solar radiation torques begin decelerating the system. (4) The system is once again locked until the desired terminal configuration is attained. Ideally, the system should reach the desired sun incidence angle with a near-zero residual angular acceleration. (5) At this time the system is unlocked and the wheels steer the bus/boom into a solar-torque equilibrium configuration at the desired sun-sail incidence angle and the system is locked until the next maneuver is needed to follow the planned trajectory. A MATLAB simulation environment was developed in order to validate the above maneuver under a variety of orbital/environmental disturbances. A control design model was first derived from the full nonlinear dynamics. A PD control law governing the time rate of change of the reaction wheel's angular momentum was designed based on the linearized dynamic equations. The resulting control law was then applied to the full nonlinear dynamics in the simulation. The motion time histories (displayed in terms of the five phases discussed above) with associated angular momentum penalty for a 30 degree slew near perigee (true anomaly = 160 [deg]) are shown in Figures 4-14. Here the absolute orientation of the bus is denoted θ_B and the orientation of the sail relative to the bus is denoted θ_S. Note that the 30 degree maneuver was performed over a total time of 1800 [sec] with a residual wheel angular momentum hw = 0.193 [N-m-s].

Equations of Motion

In this section we develop the equations of motion for the solar sail spacecraft shown in Figure 1. We assume from the outset that the solar sail is in orbit about a single gravitational primary body. However, the resulting dynamic model can be readily modified to account for the dynamics in the absence of any gravitational field. The spacecraft is idealized as the following system of three rigid bodies:

1. The sailcraft bus-boom structure is modeled as a single rigid body (identified as the basebody) B with center-of-mass (COM) C_B as shown in Figure 1.

2. The solar sail S is modeled as a rigid body* with COM at C_S. Here we assume that S is connected to the basebody B via a two degree-of-freedom (DOF) rotational joint H where H is assumed to be a ***bi-state*** device that is either free or locked.

3. The reaction wheel assembly (RWA) is modeled as a rigid rotor W. Here we assume that the COM of the RWA, denoted C_W, is attached to the base-body B at C_B via a single DOF rotational joint R.

The complete system geometry is shown in Figure 1. Note that the bi-state hinge H is offset from C_B by a length l. Further, we assume that the center-of-mass C_S of the sail is is offset from the hinge H by a small distance δ, and that the center-of-pressure (COP) of the sail (denoted C_p) is offset from C_S by a small distance ϵ along the length of the sail. In the sequel we will assume that the motion of the spacecraft is planar.

In order to facilitate the dynamic analysis a set of judiciously chosen reference frames are required. The reference frames used in this analysis are defined as follows:

- The inertial frame $\mathcal{F}_N = \{\vec{n}_1, \vec{n}_2, \vec{n}_3\}$ with origin at the center of the gravitational primary. The unit vector $\vec{n}_1$ is taken to point toward perigee, $\vec{n}_3$ is normal to the orbital plane, and $\vec{n}_2$ completes the right-handed triad.† See Figure 3.

- The orbital frame $\mathcal{F}_0 = \{\vec{o}_1, \vec{o}_2, \vec{o}_3\}$ with origin at C_B. Here the unit vector $\vec{o}_1$ points anti-nadir, $\vec{o}_3$ is normal to the orbital plane, and $\vec{o}_2$ completes the right-handed triad. The angle between $\vec{o}_1$ and $\vec{n}_1$ is the true anomaly ν. See Figure 3.

- The basebody frame $\mathcal{F}_B = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ with origin at C_B and base vectors as shown in Figure 2. The basebody frame is rigidly attached to the basebody structure and rotates with the basebody. The relative angle between $\vec{o}_1$ and $\vec{b}_1$ is denoted θ_b.

- The wheel frame $\mathcal{F}_w = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ with origin at C_W is rigidly attached to the wheel and rotates with the wheel. The angle between $\vec{w}_1$ and $\vec{b}_1$ is denoted θ_w. See Figure 2.

- The sail body frame $\mathcal{F}_{S_b} = \{\vec{s}_1, \vec{s}_2, \vec{s}_3\}$ with origin at C_S is rigidly attached to the sail and rotates with the sail. Here $\vec{s}_1$ points along the sail, $\vec{s}_2$ points anti-normal to the sail, and $\vec{s}_3$ completes the right-handed triad. The angle between $\vec{b}_1$ and $\vec{s}_1$ is denoted θ_s. See Figure 3.

- The sail surface frame $\mathcal{F}_{S_s} = \{\vec{n}, \vec{t}, \vec{b}\}$ where $\vec{n}$ is normal to the sail, $\vec{t}$ points along the sail, and $\vec{b}$ completes the right-handed triad. The origin of $\mathcal{F}_{S_s}$ is taken at the sail center of pressure C_p. See Figure 2.

The equations of motion for the multibody system described above can be generated in a number of ways [3],[4],[5],[6]. However, the formulation of the equations of motion for systems consisting of open chain topologies (i.e., systems with serial or tree-topology configurations) and holonomic joints is greatly facilitated by application of *Lagrange's Form of D'Alembert's Principle* [3],[6]:

$$Q_j + Q_j^* = 0 \quad j = 1, 2, \ldots, n \tag{1}$$

where

$$Q_j = \sum_{i=1}^{N} \vec{F}_i \cdot \vec{\gamma}_{ij} + \vec{M}_i \cdot \vec{\beta}_{ij} \tag{2}$$

denote the generalized forces due to applied forces and moments, and

$$Q_j^* = \sum_{i=1}^{N} m_i \dot{\vec{v}}_i \cdot \vec{\gamma}_{ij} + (\mathbf{I}_i \dot{\vec{\omega}}_i + \vec{\omega}_i \times \mathbf{I}_i \omega_i) \cdot \vec{\beta}_{ij} \tag{3}$$

denotes the generalized forces due to inertia forces and moments ‡. Here n denotes the number of degrees-of-freedom (DOF) of the system, N denotes the number

*The supporting truss structure of the solar sail is assumed to be stiff enough to validate this assumption. See [7] for a detailed discussion.

†For free space applications the inertial frame can be taken as any convenient inertial frame.

‡The form of the equations given here assumes that the equations of motion are developed with respect to the center of mass of each body in the multibody chain under study.

of rigid bodies in the system, the dot denotes the standard inner product, m_i is the mass of body i, $\mathbf{I_i}$ is the inertia dyadic of body i about the COM of body i, $\dot{\vec{v}}_i$ is the absolute velocity of the COM of body i, $\vec{\omega}_i$ is the absolute angular velocity of body i, $\dot{\vec{\omega}}_i$ is the absolute rate of change of the angular velocity of body i, $\vec{F}_i$ denotes the resultant of all external forces[§] acting on body i, and $\vec{M}_i$ denotes the resultant moment of all external forces and couples acting on body i. The velocity coefficients and angular velocity coefficients are given by

$$\vec{\gamma}_{ij} = \frac{\partial \vec{r}_i}{\partial q_j} \tag{4}$$

$$\vec{\beta}_{ij} = \frac{\partial \vec{\omega}_i}{\partial \dot{q}_j} \tag{5}$$

where q_j and $\dot{q}_j$ denote the j^{th} generalized coordinate and its derivative respectively,[¶] and $\vec{r}_i$ denotes the absolute position of the COM of body i in the multibody chain.

The generalized coordinates representing the configuration of the multibody solar sail system are partitioned into two classes. The first class is given by $q_o = (x, y)$ where (x, y) denotes the absolute position of C_B resolved in $\mathcal{F}_N$. The second class $q_a = (\theta_b, \theta_w, \theta_s)$ describe the orientation freedoms of the spacecraft where θ_b denotes the attitude of the basebody relative to the orbital frame, θ_w denotes the attitude of the wheel relative to the basebody, and θ_s denotes the attitude of the sail relative to the basebody. In this analysis we assume that the orbital dynamics are not influenced by the attitude dynamics. As a result, the orbital analysis can be formulated and solved independently of the attitude dynamics. The equation describing the orbital dynamics is

$$\ddot{q}_o = -\frac{\mu q_o}{|q_o|^3} \tag{6}$$

where $q_o = [x\ y]^T$, $|\bullet|$ denotes the standard norm, and μ is the gravitational parameter of the gravitational primary (i.e., Earth). Note that the orbital motion of the system is assumed pure Keplerian.

The attitude dynamics of the system are now developed by applying the following procedure to the attitude related generalized coordinates:

1. Determine $\vec{r}_i(q)$ and $\vec{\omega}_i(q, \dot{q})$ where $q = q_a$.

2. Determine the velocity and angular velocity coefficients $\vec{\alpha}_{ij}$ and $\vec{\beta}_{ij}$ via (4) and (5).

3. Determine the absolute time derivatives $\vec{v}_i = \frac{{}^N d\vec{r}_i}{dt}$, $\dot{\vec{v}}_i = \frac{{}^N d^2\vec{r}_i}{dt^2}$, and $\dot{\vec{\omega}}_i = \frac{{}^N d\vec{\omega}_i}{dt}$. Here $\frac{{}^N d}{dt}$ denotes differentiation with respect to an observer fixed in the inertial frame $\mathcal{F}_N$ [5].

4. Develop expression for the external forces $\vec{F}_i$ and moments $\vec{M}_i$ acting on each body of the system.

5. Substitute the above information into the equations of motion (1).

Following the above procedure we note from Figures 1-3 that

$$\vec{r}_1 = x\vec{n}_1 + y\vec{n}_2 \tag{7}$$

$$\vec{r}_2 = \vec{r}_1 - l\vec{b}_2 - \delta\vec{s}_2 \tag{8}$$

$$\vec{r}_3 = \vec{r}_1 \tag{9}$$

and

$$\vec{\omega}_1 = (\dot{\theta}_b + \dot{\nu})\vec{n}_3 \tag{10}$$

$$\vec{\omega}_2 = (\dot{\theta}_b + \dot{\theta}_s + \dot{\nu})\vec{n}_3 \tag{11}$$

$$\vec{\omega}_3 = (\dot{\theta}_b + \dot{\theta}_w + \dot{\nu})\vec{n}_3 \tag{12}$$

where the subscripts $(1, 2, 3)$ denote bodies B, S, and W respectively. Recall from above that the orbital variables (x, y) (and hence ν) and their associated time derivatives are taken as prescribed in the attitude analysis[‖].

The nonzero velocity coefficients $\vec{\gamma}_{ij}$ are given by

$$\vec{\gamma}_{21} = l\vec{b}_1 + \delta\vec{s}_1 \tag{13}$$

$$\vec{\gamma}_{22} = \delta\vec{s}_1 \tag{14}$$

In a similar fashion the nonzero angular velocity coefficients are given by

$$\vec{\beta}_{11} = \vec{\beta}_{21} = \vec{\beta}_{22} = \vec{\beta}_{31} = \vec{\beta}_{33} = \vec{n}_3 \tag{15}$$

Once the velocity coefficients have been determined the absolute linear and angular accelerations of

[§]It can be shown that all workless internal reaction forces are automatically eliminated from the dynamic analysis when Lagrange's form of D'Alembert's Principle is employed.

[¶]For systems involving quasi-coordinates and/or nonholonomic joints the generalized coordinates q_j and their time derivatives $\dot{q}_j$ must be replaced in the definition of the velocity and angular velocity coefficients with generalized speeds u_j and their time derivatives $\dot{u}_j$. Under these conditions Lagrange's form of D'Alembert's Principle must be modified and is commonly called Kane's Form of D'Alembert's Principle [5].

[‖]The true anomaly v and its associated time derivatives are known once (x, y) and their associated time derivatives are known.

each body are computed by differentiating (7)-(12) in $\mathcal{F}_N$:

$$\begin{aligned} \dot{\vec{v}}_1 &= \ddot{x}\vec{n}_1 + \ddot{y}\vec{n}_2 && (16) \\ \dot{\vec{v}}_2 &= \dot{\vec{v}}_1 + l(\ddot{\nu} + \ddot{\theta}_b)\vec{b}_1 + l(\dot{\nu} + \dot{\theta}_b)^2\vec{b}_2 + \delta(\ddot{\nu} \\ &\quad +\ddot{\theta}_b + \ddot{\theta}_s)\vec{s}_1 + \delta(\dot{\nu} + \dot{\theta}_b + \dot{\theta}_s)^2\vec{s}_2 && (17) \\ \dot{\vec{v}}_3 &= \dot{\vec{v}}_1 && (18) \end{aligned}$$

and

$$\begin{aligned} \dot{\vec{\omega}}_1 &= (\ddot{\theta}_b + \ddot{\nu})\vec{n}_3 && (19) \\ \dot{\vec{\omega}}_2 &= (\ddot{\theta}_b + \ddot{\theta}_s + \ddot{\nu})\vec{n}_3 && (20) \\ \dot{\vec{\omega}}_3 &= (\ddot{\theta}_b + \ddot{\theta}_w + \ddot{\nu})\vec{n}_3 && (21) \end{aligned}$$

The external forces acting on each body are

$$\begin{aligned} \vec{F}_1 &= \vec{f}_{g1} && (22) \\ \vec{F}_2 &= \vec{f}_{sp} + \vec{f}_{g2} && (23) \\ \vec{F}_3 &= \vec{f}_{g3} && (24) \end{aligned}$$

where

$$\vec{f}_{gi} = -\frac{\mu\vec{r}_i}{r_i^3} \quad (25)$$

denotes the resultant gravitational force at the COM of body i, and $\vec{f}_{sp}$ denotes the solar pressure force generated on the solar sail at its center-of-pressure (COP). Here the sail is modeled as a flat rectangular plate. (See Figure 2.) The expression for the solar radiation force on a flat plate is well-known [2],[7],[8]:

$$\begin{aligned} \vec{f}_{sp} &= f_n\vec{n} + f_t\vec{t} && (26) \\ &= -f_t\vec{s}_1 - f_n\vec{s}_2 && (27) \end{aligned}$$

where

$$\begin{aligned} f_n &= PA\{(1+\sigma_{rs})\cos^2\alpha + \frac{2}{3}\sigma_{rd}\cos\alpha\} && (28) \\ f_t &= PA(1-\sigma_{rs})\cos\alpha\sin\alpha && (29) \end{aligned}$$

Here $P = 4.563 \times 10^{-6}\frac{N}{m^2}$ is the nominal value of solar radiation pressure at 1 AU, A denotes the sail surface area, σ_{rs} denotes the coefficient of specular reflection, σ_{rd} is the coefficient of diffuse reflection, and $\cos(\alpha) = \vec{n} \cdot \hat{I}$ denotes the sun incidence angle between the sail normal ($\vec{n} = -\vec{s}_2$) and the incident solar radiation direction $\hat{I}$. (See Figure 2.) Note that $\alpha = \nu + \theta_b + \theta_s$. We also assume that the sun-incidence direction $\hat{I}$ is fixed in the inertial frame.

The external moments acting on the system are given by

$$\begin{aligned} \vec{M}_1 &= (\tau_w + \tau_{g1} + \tau_f)\vec{n}_3 && (30) \\ \vec{M}_2 &= (\tau_{sp} + \tau_{g2} - \tau f)\vec{n}_3 && (31) \\ \vec{M}_3 &= -\tau_w\vec{n}_3 && (32) \end{aligned}$$

where

$$\tau_{gi} = \frac{3\mu}{r_i^3}(I_{yy}^i - I_{xx}^i)o_{xi}o_{yi} \quad (33)$$

is the gravity gradient torque exerted on body i. Here I_{yy}^i and I_{xx}^i denote the principal moments of inertia of body $i = (B, S, W)$ about the x and y axes of $\mathcal{F}_i$, and o_{xi} and o_{yi} denote the components of $-\vec{o}_1$ in $\mathcal{F}_i$. Also

$$\tau_f = c_1 sgn(\dot{\theta}_s) + c_2\dot{\theta}_s \quad (34)$$

denotes Coulomb-viscous friction at the bi-state hinge H, τ_w denotes the torque applied by the wheel to the basebody, and $\tau_{sp} = \epsilon f_n$ denotes the solar pressure torque generated on the sail.

The equation of motion corresponding to the θ_b coordinate is

$$\begin{aligned} &(I_b + m_s l^2)\ddot{\theta}_b + (I_b + m_s l^2)\ddot{\nu} + I_s(\ddot{\theta}_b + \ddot{\theta}_s + \ddot{\nu}) + \dot{h}_w \\ &\quad +m_s l\{\ddot{x}\cos(\theta_b + \nu) + \ddot{y}\sin(\theta_b + \nu)\} + g_1(\delta) \\ &= \\ &\tau_{g1} + \tau_{g2} + \tau_{sp} - f_t\delta - f_t l\cos\theta_s + f_n l\sin\theta_s \\ &\quad -\frac{m_s\mu l}{(x_s^2+y_s^2)^{\frac{3}{2}}}\{x_s\cos(\theta_b + \nu) + y_s\sin(\theta_b + \nu)\} + \tau(\delta) \\ &&& (35) \end{aligned}$$

where

$$\begin{aligned} g_1(\delta) \;:=\; & m_s l\{\delta(\ddot{\theta}_b + \ddot{\theta}_s + \ddot{\nu})\cos\theta_s \\ & -\delta(\dot{\theta}_b + \dot{\theta}_s + \dot{\nu})^2\sin\theta_s\} \\ & +m_s\delta\{\ddot{x}\cos(\theta_b + \theta_s + \nu) \\ & +\ddot{y}\sin(\theta_b + \theta_s + \nu) \\ & +l(\dot{\nu} + \dot{\theta}_B)^2\sin\theta_s \\ & +\delta(\ddot{\theta}_b + \ddot{\theta}_s + \ddot{\nu}) + l(\ddot{\nu} + \ddot{\theta}_b)\cos\theta_s\} \quad (36) \end{aligned}$$

and

$$\begin{aligned} \tau(\delta) \;:=\; & \frac{-m_s\mu\delta}{(x_s^2 + y_s^2)^{\frac{3}{2}}}\{x_s\cos(\theta_b + \theta_s + \nu) \\ & +y_s\sin(\theta_b + \theta_s + \nu)\} \quad (37) \end{aligned}$$

Note that both g_1 and τ are zero when evaluated at $\delta = 0$. In the above derivation we have used the fact that the absolute time rate of change of the wheel's angular momentum is given by

$$\dot{h}_w = I_w(\ddot{\theta}_b + \ddot{\theta}_w + \ddot{\nu}) = -\tau_w \quad (38)$$

In the next section we will develop a control strategy based on $\dot{h}_w$.

The equations of motion corresponding to θ_s and θ_w are

$$\begin{aligned} I_s(\ddot{\theta}_b + \ddot{\theta}_s + \ddot{\nu}) + g_2(\delta) &= \tau_{sp} + \tau_{g2} - \tau_f \\ &\quad -f_t\delta + \tau(\delta) && (39) \\ \dot{h}_w &= -\tau_w && (40) \end{aligned}$$

respectively. Here

$$
\begin{aligned}
g_2(\delta) \quad := \quad & m_s\delta\{\ddot{x}\cos(\theta_b+\theta_s+\nu) \\
& +\ddot{y}\sin(\theta_b+\theta_s+\nu) \\
& +l(\dot{\nu}+\dot{\theta}_B)^2\sin\theta_s+\delta(\ddot{\theta}_b+\ddot{\theta}_s+\ddot{\nu}) \\
& +l(\ddot{\nu}+\ddot{\theta}_b)\cos\theta_s\} \qquad (41)
\end{aligned}
$$

where $h_w = I_w(\dot{\theta}_b+\dot{\theta}_w+\dot{\nu})$. Note that $g_2 = 0$ when $\delta = 0$.

To recapitulate, the equations of motion of the orbiting multibody solar sail are given by (6),(35),(39), and (40). The orbital dynamics (6) are pure Keplerian and can be solved independently of the attitude motion. The resulting time histories of the orbital variables (x,y,ν) and their associated time derivatives are then treated as prescribed in the attitude motion equations (35),(39), and (40). We comment that the above dynamic model has been developed with the assumption that the bi-state gimbal is free. However, the locked-gimbal equations of motion follow immediately from the free-gimbal equations by

1. Omitting the θ_s dynamic equation (39) from the analysis.

2. Setting the hinge angle to its locked value $\theta_s = \theta_{sl}$ and then letting $\dot{\theta}_s = \ddot{\theta}_s = 0$ in the remaining attitude equations (35) and (40). The orbital motion equations remain unaltered.

Note that the attitude equations of motion (of the free or locked system) can be modified for deep space applications by setting the true anomaly v and its time derivatives equal to zero in the appropriate equations.

Control Design and Analysis

The multibody dynamic model developed in the previous section is now utilized to demonstrate that an orbiting solar sail with a bi-state gimbal architecture and bus-mounted reaction wheels can realize a large commanded sail-sun incidence angle α_d with a minimal wheel momentum storage penalty and negligible CSI with the sail and booms. This is in contrast to performing large angle turns via continuous actuation directly at the gimbal H (e.g., via stepper motors which excite the sail/boom dynamics.)** Direct motor torques injected at the gimbal can produce unwanted vibrations of the solar sail, and thrust vector errors. Our articulation and control stragety is based on the following five phase maneuver (See Figures 4 - 14, and 16 - 19):

- Phase 1: Beginning with the system in the initial configuration $\theta_b(0) = \theta_s(0) = 0$, the reaction wheel is commanded to create a small center-of-mass to center-of-pressure (CM-CP) offset ($\approx 5°$) while the bi-state gimbal is free. This is accomplished by articulating the bus/boom relative to the solar sail.

- Phase 2: Once the appropriate CM-CP offset is attained, the bi-state gimbal is locked and the spacecraft begins accelerating toward the desired sun-incidence angle via solar radiation torque. The system undergoes free (uncontrolled) motion during this phase. Recall that the system is also subjected to gravity gradient disturbance torques whose magnitude depend on the inverse cube of the distance between the spacecraft and the central body. As a result, the solar pressure radiation torques should be at least an order of magnitude greater than other disturbance torques acting on the system during this phase.

- Phase 3: At an appropriate switching time t^*, the bi-state gimbal is again unlocked and the wheel is used to reverse the polarity of the CMCP offset (by again articulating the bus-boom relative to the sail) and hence reverse the direction of the solar radiation torque. As a result, solar radiation torques begins decelerating the system. The switching time t^* is predicted off-line by utilizing a linear model of the free-motion of the system along with knowledge of the desired sun-incidence angle α_d.

- Phase 4: The hinge is once again locked until the commanded terminal configuration is attained. Ideally, the system should reach the desired sun incidence angle with a near-zero residual angular acceleration. Again, the system undergoes free motion during this phase. The solar radiation torques must be at least an order of magnitude greater than any other disturbance torques during this phase as well.

- Phase 5: Once the system has reached the commanded sun incidence angle the hinge is unlocked. At this time the RWA steers the bus/boom into a solar-torque equilibrium configuration and the bi-state hinge is again locked. At this point, the attitude of the system will be under reaction wheel control in all axes.

**This technique also requires bus-mounted reaction wheels in addition to the gimbal stepper motors.

Note that RWA control torques are required during Phases 1,3, and 5. Moreover, most of the wheel momentum injected into the bus-sail system during Phase 1 is automatically unloaded during Phases 3 and 5.

The design of a proportional-derivative (PD) RWA attitude control law for articulating the bus-boom relative to the sail during the controlled phases (i.e., Phases 1,3, and 5) is now discussed. First, the full nonlinear multibody model developed in the previous section is linearized to create a model suitable for developing a linear compensator. However, all simulation results presented in the sequel are based on applying the resulting linear control law to the *full nonlinear plant*[††] given by equations (35),(39), and (40).

The attitude controller design model is derived from the full nonlinear multibody model by making the following approximations in (6) and (35):

1. $\delta \approx 0$
2. Due to the fact that $\vec{r}_1 \approx \vec{r}_2$ we assume that $(x_s, y_s) \approx (x, y)$

Under these simplifications we find after some manipulation

$$\begin{aligned}(I_b + m_s l^2)\ddot{\theta}_b + I_s(\ddot{\theta}_b + \ddot{\theta}_s) &= f_n l \theta_s - \dot{h}_w \\ &\quad + \tau_{d1} \qquad (42)\\ I_s(\ddot{\theta}_b + \ddot{\theta}_s) &= \tau_{d2} \qquad (43)\\ \dot{h}_w &= -\tau_w \qquad (44)\end{aligned}$$

where τ_{d1} and τ_{d2} represent the lumped effect of all disturbances (friction, orbital motion, solar torques, etc.) on the spacecraft during the controlled phases. It follows from (42)-(44) that the transfer function relating the input RWA torque τ_w to the solar sail angle θ_s is given by

$$\frac{\bar{\theta}(s)}{\bar{\tau_w}(s)} = \frac{-1}{(I_b + m_s l^2)s^2 + f_n l} \qquad (45)$$

where s denotes complex frequency and the overbar denotes the Laplace transformation of the given signal. The linear dynamics (45) are then utilized to design a PD compensator of the form

$$\tau = -K_p(\theta_b - \theta_b^d) - K_d(\dot{\theta}_b - \dot{\theta}_b^d) \qquad (46)$$

valid for the free hinge modes of operation. Here θ_b^d and $\dot{\theta}_b^d$ denote the desired sail attitude and rate profiles respectively. It is easily shown that the closed loop transfer function based on the linear model is stable if $K_p > 0$ and $K_d > 0$.

A simulation environment was constructed to test the feasibility of commanding the orbiting solar sail to a large sun-incidence angle α_d. The particular orbit of interest in our study is representative of the proposed NASA-JPL New Millennium Program Space Technology solar sail flight experiment that involves a sail in a Super-Synchronous-Transfer-Orbit (SSTO). The SST0 is a highly elliptical Earth orbit with eccentricity = 0.8196, perigee = 8375 [km], apogee = 84482 [km], and inclination = 35 [deg]. (See Figure 3.)

The nominal values of the parameters used in the simulation were taken as $m_b = 120[kg]$, $m_s = 40[kg]$, $I_b = 20[kg{\cdot}m^2]$, $I_s = 3000[kg{\cdot}m^2]$, $I_w = 10[kg{\cdot}m^2]$, $l = 2[m]$, $\epsilon = 0.1[m]$, $\delta = 0.5[m]$, $A = 1500[m^2]$, $\rho_s = 0.83$, and $\rho_d = 0.15$. The friction parameters are $c_1 = 0.0078[N \cdot m]$ and $c_2 = 2.7e-3[N \cdot m\backslash rad\backslash sec]$. The solar pressure P at 1AU is $4.563 \times 10^{-6}[\frac{N}{m^2}]$. The maximum torque the wheel can deliver is $\pm 0.14[N \cdot m]$ and its maximum momentum storage capability is $\pm 4[N \cdot m \cdot sec]$.

The system response (broken down in terms of the five phase sequence discussed above) with associated angular momentum penalty for a 30 degree slew (i.e., $\alpha_d = 30[deg]$) near perigee ($\nu = 160$ [deg]) are shown in Figures 4-14. Note that the 30 degree maneuver was performed over a total time of 1800 [sec] with a residual wheel angular momentum $h_W = 0.193$ [N-m-s].

As mentioned above, the relative magnitude between the solar pressure and gravity gradient torques is of paramount importance for the success of any proposed solar pressure control strategy. For example, it was discovered that the maneuver could not be performed near perigee using only solar torques (even utilizing a much larger commanded CMCP offset in Phase 1) because the gravity gradient torques were dominant *and* opposite in direction to the solar radiation torques. However, when our concept and methodology is applied to realize solar sail maneuvers in Earth orbital applications above 30,000 Km or in deep space, no such issues will arise [9].

Acknowledgement

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[††] All parameters occurring in the the truth model are also assumed to have 5 percent uncertainty from their nominal values.

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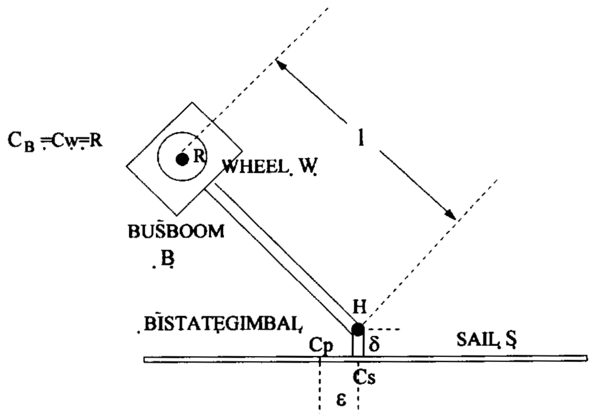

Figure 1: System Geometry

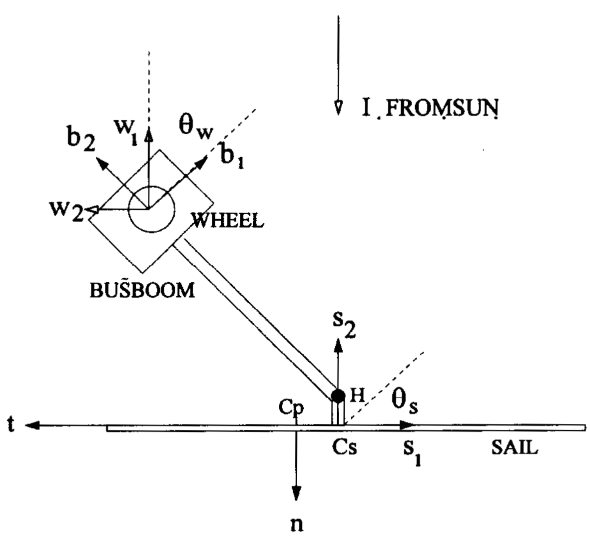

Figure 2: Reference Frames

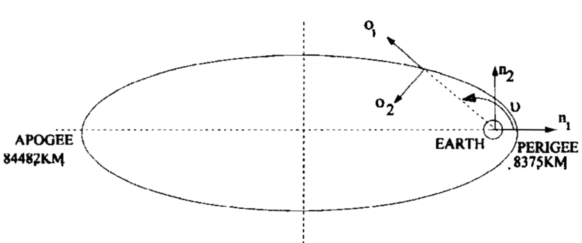

Figure 3: Super-Synchronous Transfer Orbit

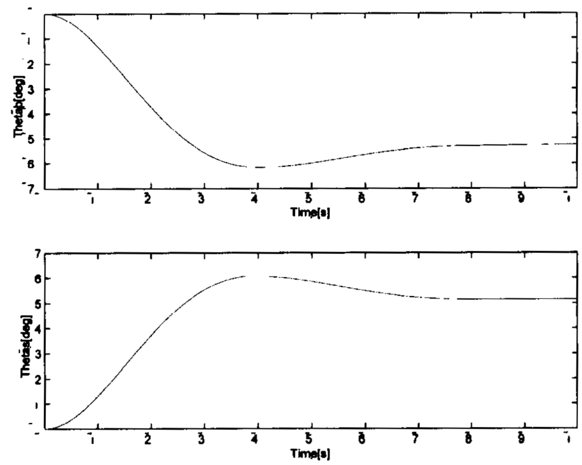

Figure 4: Phase1: Bus Inertial Attitude and Sail-Bus Relative Attitude

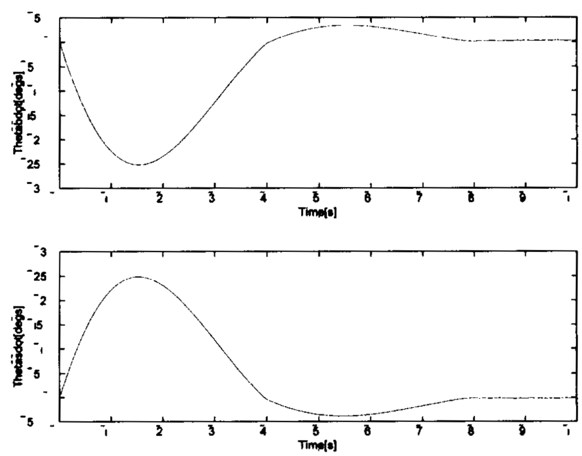

Figure 5: Phase1: Bus Absolute Rate and Sail-Bus Relative Rate

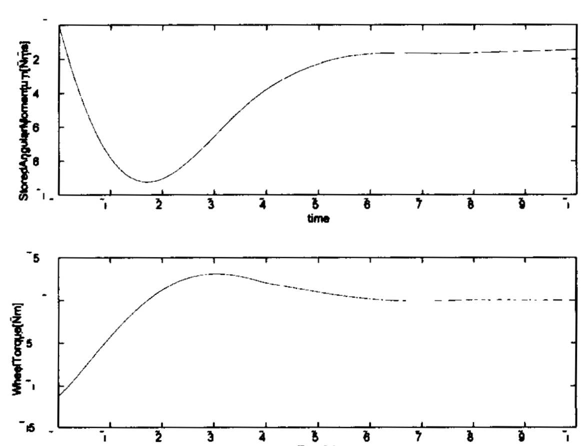

Figure 6: Phase1: Associated RWA Torques and Wheel Angular Momentum

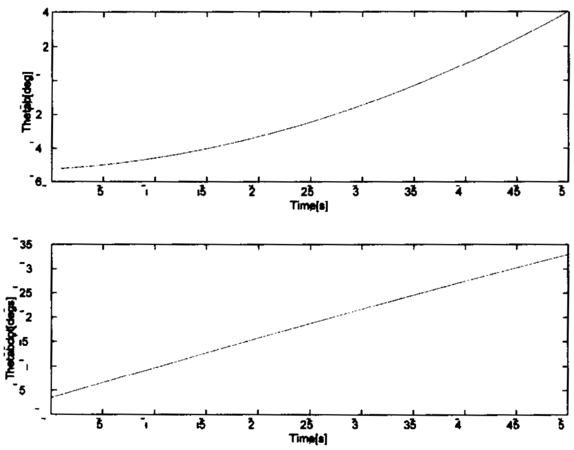

Figure 7: Phase2: Locked System Absolute Orientation and Rate

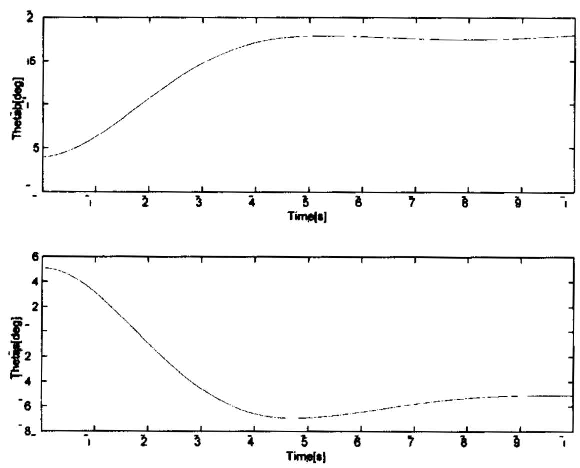

Figure 8: Phase3: Bus Inertial Attitude and Sail-Bus Relative Attitude

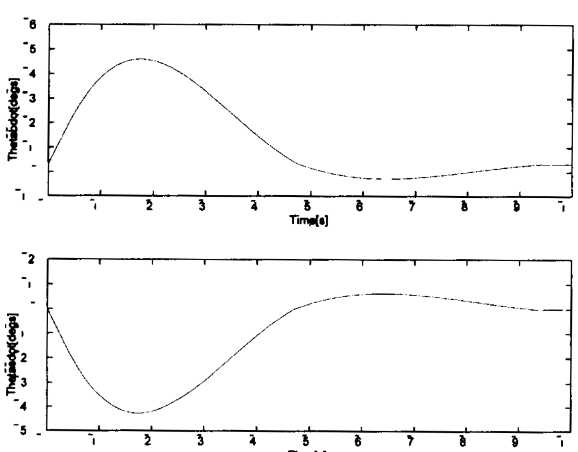

Figure 9: Phase3: Bus Absolute Rate and Sail-Bus Relative Rate

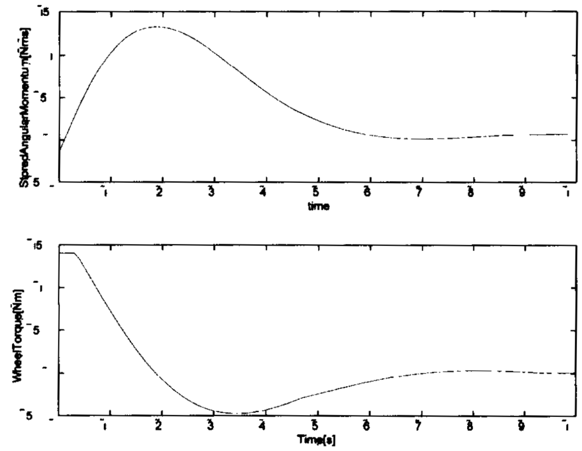

Figure 10: Phase3: Associated RWA Torques and Wheel Angular Momentum

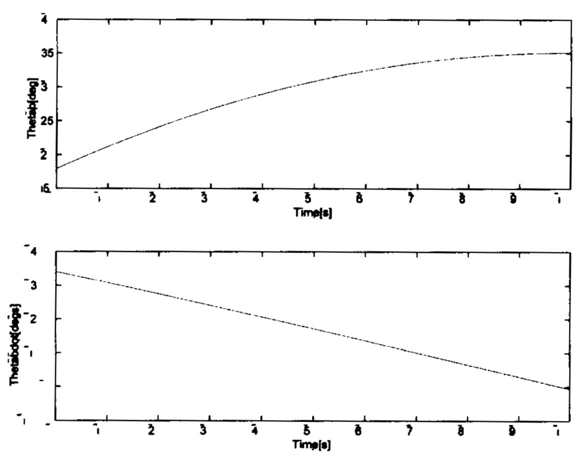

Figure 11: Phase4: Locked System Absolute Orientation and Rate

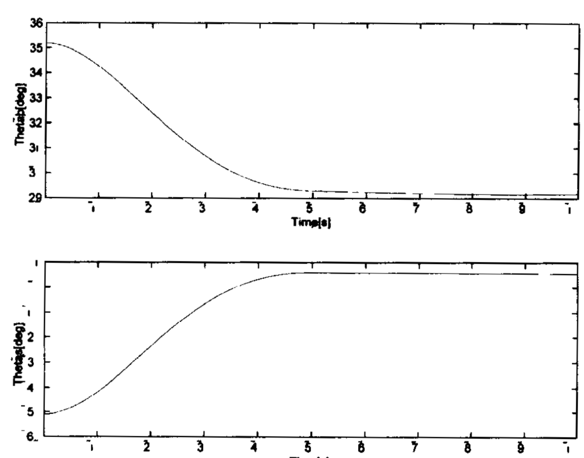

Figure 12: Phase5: Bus Inertial Attitude and Sail-Bus Relative Attitude

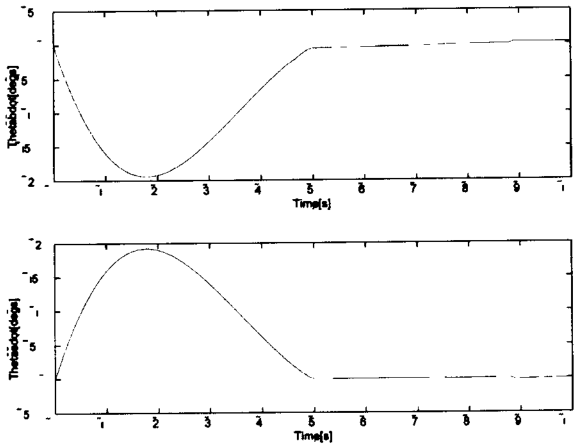

Figure 13: Phase5: Bus Absolute Rate and Sail-Bus Relative Rate

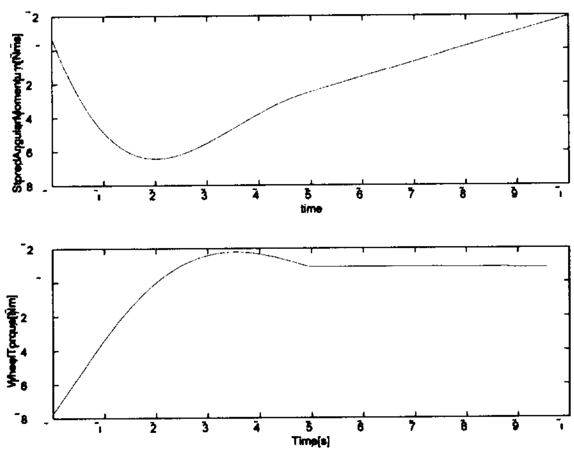

Figure 14: Phase5: Associated RWA Torques and Wheel Angular Momentum

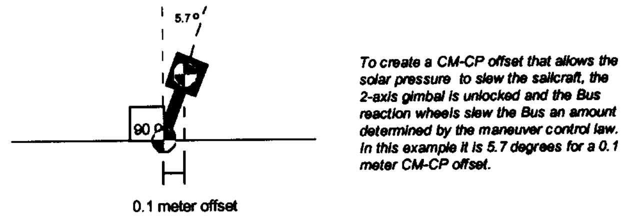

Figure 15:

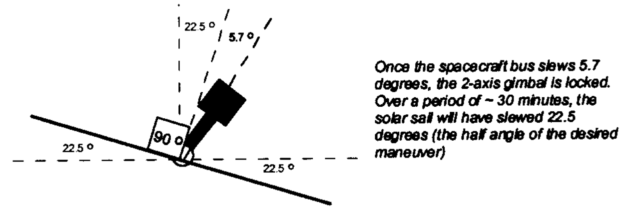

Figure 16:

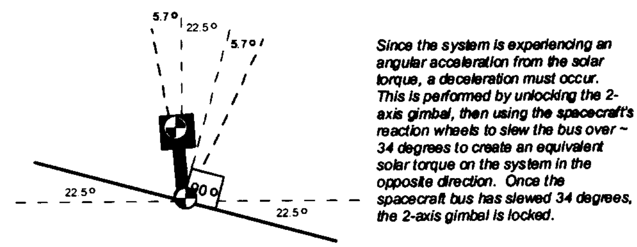

Figure 17:

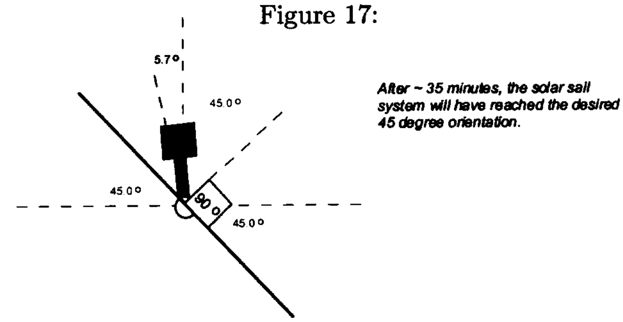

Figure 18:

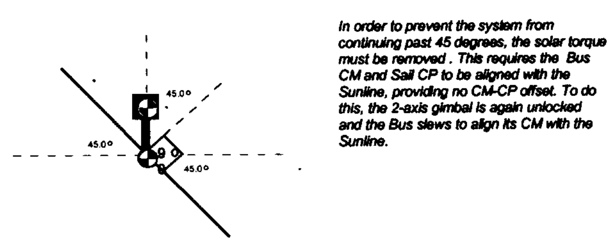

Figure 19: